Climate Models: The Energy Balance Model

This is a self-guided (group) activity. In this module, we will explore a simple climate model: the energy balance model.

The energy balance model predicts the latitudinal distribution of the earth's surface temperature, T. This model is based on the concept that the energy the earth receives from the sun's radiation needs to balance the radiation the earth loses to space by reemission at its temperature, T.

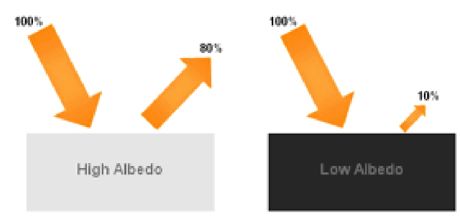
0.1 Background

Set-up: In order to measure the latitudinal distribution of the earth's surface temperature, there are many different factors we need to take into consideration:

- incoming solar radiation,
- albedo,
- outward radiation,
- ice dynamics,
- transport of heat.

Each of these factors can cause great changes to the earth's surface temperature, and in this module we will see how.

Example: We need to take into account the reflection of solar radiation back to space by the ice, snow, and clouds. This is called the **albedo** effect.



To get an idea of how drastic of a difference the albedo effect makes, listen to this: ice-free models give a temperature of $46°C$ at the equator and $-43°C$ at the pole, whereas our current model gives a temperature of $27°C$ at the equator and $-13°C$ at the pole.

Taking into the consideration the presence of ice makes the problem of modeling the latitudinal distribution of the earth's surface temperature much more interesting and accurate:

- Ice (white surfaces in general, like snow and clouds) reflects sunlight back to space more than land or ocean surfaces do, thus, the earth is actually losing more heat (with less absorption of the sun's radiation) than if there was no ice cover. Therefore, it gets colder and thus, more ice forms and the ice sheet advances towards the equator. The ice sheets moving toward the equator help reflect more solar radiation away from the earth, thus __________ (cooling down/heating up) the temperature near the equator.
- On the other hand, if the solar radiation is increased, then ice melts a little near the ice edge and thus, exposes more land, which absorbs more solar radiation, which makes the earth warmer, and more ice melts, so the ice sheet retreats towards the poles. The ice sheets moving away from the equator allow for more absorption of solar radiation near the equator, thus __________ (cooling down/heating up) the temperature near the equator.

Q1: How would the earth's ice-free surface temperature compare to the earth's ice-covered temperature? Why?

0.2 Incoming Solar Radiation

Solar energy input: The incoming solar radiation at the top of the earth's atmosphere, i.e., the rate of solar energy input per unit earth area, is $Qs(y)$, where

- $y = \sin\theta$ with θ being the latitude
  (observe that since $y = \sin\theta$, it can only take values between 0 and 1)
- Q is the overall (integrated) total solar input into the atmosphere-ocean system
  (the magnitude of Q is 343 watts per square meter (W/m^2) at present),
- $s(y)$ is called the latitudinal distribution function and is normalized so that $\int_0^1 s(y)dy = 1$.

The total solar energy input is obtained by integrating over the surface of the earth.

Q2: What is the total solar energy input for the earth of radius a?

$$\int_{-\pi/2}^{\pi/2} Q \cdot s(\sin\theta) 2\pi a \cos\theta a d\theta =$$

The integrated solar energy input should be equal to the solar flux (the measure of how much light energy is being radiated in a given area, W/m^2) intercepted by an area of the circular disk of the earth seen by the sun: $S \cdot \pi a^2$, where S is the solar constant.

Q3: What is Q in terms of S?

The function $s(y)$ is uniformly approximated to be $s(y) = 1 - S_2 P_2(y)$, where $S_2 = 0.482$ and $P_2(y) = (3y^2 - 1)/2$ for the current obliquity (angle Earth's axis of rotation is tilted as it travels around the sun) of the earth's orbit.

Q4: Using the uniform approximation of $s(y)$, calculate $s(y)$ for the following values of y:

1. $y = 0.25$:
2. $y = 0.5$:
3. $y = 0.75$:

Recap: We have that:

- $Qs(y) =$
- $Q =$
- $s(y) =$
- $y =$
- $\theta =$

0.3 Albedo

Recall that, in short, **albedo** is the solar radiation that is reflected from ice (or snow, etc) on the earth's surface back to space. The albedo function, $\alpha(y)$ is the fraction of the solar radiation that hits the earth and is reflected back into space without being absorbed at all by the earth.

Q5: How can we express the amount of solar radiation absorbed by the earth per unit area?

Hint: $Qs(y) =$ solar radiation absorbed by the earth per unit area, and $\alpha(y) =$ a fraction (meaning between 0 and 1) of the solar radiation that is not absorbed by the earth per unit area.

Q6: How does more albedo change the net amount of solar radiation absorbed by the earth? Why?

Q7: How does less albedo change the net amount of solar radiation absorbed by the earth? Why?

Recap: We have that:

- the net solar radiation absorbed by the earth =

0.4 Outward radiation

The absorbed solar energy is balanced at each latitude by reemission from earth to space and the transport of heat by the atmosphere-ocean system from this latitude to another latitude.

Let $I(y)$ be the rate of energy emission by earth per unit area. Notice that $I(y)$ is temperature dependent. The warmer the planet is, the ________________ (higher/lower) its rate of energy emission.

We have that $I = A + BT$, where T is the earth's surface temperature (in $°C$), and A, B are constants chosen empirically based on the present climate. The values are $A = 202W/m^2$ and $B = 1.90W/m^2/°C$.

Recap: We have that:

- $I(y) =$

0.5 Ice dynamics

Recall that water freezes, i.e., ice forms, when the temperature is below $0°C$. In order for permanent glaciers to be sustained, the annually averaged temperature needs to be much colder than $0°C$, especially over the oceans.

If the annually averaged temperature is exactly $0°C$, then that means that at some point during the year, the temperature is ________________ (above/below) $0°C$ and the ice melts. We say that an ice sheet forms when $T < T_C = -10°C$, where T is the earth's surface temperature and T_C is the temperature at the ice boundary.

Let y_s be the location of the ice boundary, so that towards the pole of this location (latitude) the earth is covered in ice and towards the equator of this location (latitude) the earth is ice-free. In other words, y_s is the boundary of the ice sheet, meaning that on one side of the boundary there is ice and on the other side there is no ice.

Since albedo is ________________ (higher/lower) in the ice covered part of the earth, we say that:

$$\alpha(y) = \begin{cases} \alpha_1 = 0.32 & \text{if } y < y_s, \\ \alpha_2 = 0.62 & \text{if } y > y_s. \end{cases}$$

So at the ice boundary, the temperature is taken to be $T(y_s) = T_C$. We assume that the albedo at the ice boundary is the average of the temperature on the ice side and on the ice-free side.

Q8: What is the albedo at the ice boundary, $\alpha(y_s)$?

0.6 Transport

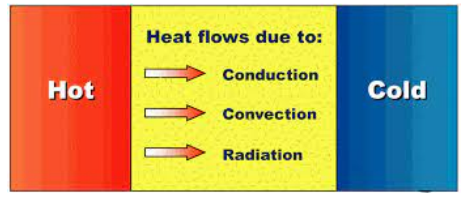
When a hot liquid is placed next to a cold liquid, the heat is often exchanged from one liquid to the other so that the temperature difference decreases, i.e., the temperatures average out. Heat is transported by the earth's atmosphere-ocean system in a number of different ways. These transport processes are lumped into a simple relaxation term for the rate of change of heat energy.

Transport: The rate of change of heat energy due to all dynamical transport processes is:

$$D(y) = C(\bar{T} - T),$$

where $\bar{T}$ is the globally averaged temperature and $C = 1.6B$ (with $B = 1.90W/m^2/°C$ from earlier).

The function $D(y)$ satisfies the constraint that transport only moves heat from hot to cold while having no effect on the globally integrated temperature. If the local temperature at a particular latitude is greater than the global mean, then the latitude will ____________ (lose/gain) heat. If the local temperature at a particular latitude is colder than the global mean, then the latitude will ____________ (lose/gain) heat.



Recap: We have that:

- $D(y) =$
- $\bar{T} =$
- $C =$

0.7 The model equation

Let $R\frac{d}{dt}T$ denote the rate of change of the earth's surface temperature, T. The parameter R is the heat capacity of the earth, which is mostly determined by the heat capacity of the atmosphere and oceans. For now, we will not worry about R.

Model equation: Our model should say that

$$R\frac{d}{dt}T = F(T),$$

where

$$\begin{aligned} F(T) = &\text{(net absorption of solar energy input)} \\ &- \text{(earth's outward radiation (i.e., the rate of energy emission by earth))} \\ &+ \text{(heat gained or lost from transport (i.e., the rate of change of heat energy))}. \end{aligned}$$

Q9: What is the equation for $F(T)$?

(Since T depends on y and t, the function $F(T)$ depends on y and t as well.)

Q10: How does an increase/decrease in the earth's outward radiation effect $F(T)$?

Q11: How does an increase/decrease in the rate of change of heat energy effect $F(T)$?

On an annual mean basis, what we are trying to calculate is symmetric about the equator, so we only need to consider the case of $y \geq 0$. After we assume this symmetry condition across the equator, we have $dT/dy = 0$ at $y = 0$. Under this condition, we have that the globally averaged temperature, $\bar{T}$, is the same as the hemispherically averaged temperature:

$$\bar{T} = \int_0^1 T(y)dy.$$

If we integrate $R\frac{d}{dt}T = F(T)$ hemisphereically, we get an equation governing the evolution of the global mean temperature:

$$R\frac{d}{dt}\bar{T} = Q(1 - \bar{\alpha}) - A - B\bar{T},$$

where Q, A, B are as defined before and

$$\begin{aligned}\bar{\alpha} &= \int_0^1 s(y)\alpha(y)dy \\ &= \int_0^{y_s} s(y)\alpha(y)dy + \int_{y_s}^1 s(y)\alpha(y)dy\end{aligned}$$

Q12: How can we simplify the integrals above ($\bar{\alpha}$) in terms of α_1 and α_2?

So we have that $\bar{\alpha} = \alpha_1$ in an ice-free earth model, and $\bar{\alpha} = \alpha_2$ in an ice-covered earth model.

For a **partially ice-covered** earth model, with the ice boundary at y_s, we have that:

$$\bar{\alpha}(y_s) = \alpha_2 + (\alpha_1 - \alpha_2)y_s[1 - 0.241(y_s^2 - 1)].$$

Q13: The present ice boundary is at $y_s = 0.95$ (corresponding to $72°N$). What is $\bar{\alpha}(y_s)$ for the present ice boundary? (Recall that $\alpha_1 = 0.32$ and $\alpha_2 = 0.62$.)

Q14: Is this closer to the ice-free albedo, α_1, or ice-covered albedo, α_2?

Q15: Find the equilibrium solution T^* of the equation of $F(T)$ by setting its right hand side to zero. Write out each part of the equilibrium solution, i.e., $I = A + BT^*$ instead of just I.

Q16: Find the global mean temperature at equilibrium. (Hint: you can find the global mean temperature at equilibrium by setting the global mean temperature equation equal to zero and solving for $\overline{T^*}$.)

Q17: Using the global mean temperature at equilibrium, $\overline{T^*}$, and the equation for the equilibrium solution, T^*, substitute the equation for $\overline{T^*}$ (from **Q16**) when it appears in $F(T^*)$ (from **Q15**) and algebraically work out the equation for the equilibrium solution.

Q18: What does the equilibrium solution in **Q17** tell us?